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A Framework for the Statistical Analysis of Probability of Mission Success Based on Bayesian Theory

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ABSTRACT

The Mission Success Prediction Capability (MSPC) is a Model-Based Systems Engineering (MBSE) approach to mission planning, used for the analysis of complex systems of precision strike (air-to-surface) weapons. This report focuses on developing a statistical method for evaluating the probability of mission success in these situations. Bayesian networks are used as a framework for the calculation of the probability in question, and a general methodology for the evaluation is formed and demonstrated in this report.

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A Framework for the Statistical Analysis of Probability of Mission Success Based on Bayesian Theory

Executive Summary

The Mission Success Prediction Capability (MSPC) is a Model-Based Systems Engineering (MBSE) approach to mission planning, used for the analysis of complex systems of precision strike (air-to-surface) weapons. This report focuses on developing a statistical method for evaluating the probability of mission success in these situations, from the type of model used in evaluation to the method for the implementation of this model.

Research has been conducted to assess which method of calculation of probability is best suited for this capability. By comparing four valid models with respect to the project and its needs, Bayesian networks have been chosen as the most appropriate statistical method for this task. This is due to many reasons, including the fact that Bayesian inference is far more capable of accounting for non-holistic and sometimes subjective data than other methods, such as logistic regression.

While it is sometimes possible, and may be necessary in future, to statistically assess the dependencies between the events in a system, expert opinion was used in this report to generate an appropriate demonstrative network for the probability of mission success.

A general formula was derived for the calculation of this probability based on the Bayesian network constructed. From this, a Matlab® function has been developed to demonstrate how this formula could be implemented in regard to the system constructed earlier. False probability distributions are used in this function for demonstration purposes. The Matlab® code gives a general method for transferring the network to a program that can calculate the probability of mission success. Software specifically designed for Bayesian networks, AgenaRisk®, has also been used to demonstrate this formula. A major issue with any probabilistic model is the computational burden and methods for avoiding and ameliorating this problem are discussed in this report.

This report covers the initial consideration of the statistical aspects of this project. From here, data needs to be collated and probability distributions generated to apply this network in practice. A method for measuring the precision of the result must also be produced to give credibility to the calculated mission success probability.

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1. Introduction

The Mission Success Prediction Capability (MSPC) is a Model-Based Systems Engineering (MBSE) approach to mission planning. It focuses on holistically analysing complex systems, more specifically those of precision strike (air-to-surface) weapons. The aim of the MSPC is to [1]:

"Use all available data to predict the success of a given mission enabling Commanders the flexibility to make more efficient and effective use of complex weapon systems."

The output of this capability is intended to be a software system which combines all the available data to simulate the proposed mission.

This report covers the initial consideration of the statistical aspects of the capability, in which a framework for measuring the probability of mission success of a weapon is developed. This includes discussion on the type of analysis method best implemented for the MSPC. The merits of the preferred method applied to analyse the probability of mission success, a Bayesian network, are discussed in the literature review in Section 2, specifically in comparison to three other methods. The other methods, which are both probabilistic and non-probabilistic, are briefly explained and their qualities relevant to the project are discussed.

The methodology of the report covers the statistical development of the Bayesian network. This includes the construction of the physical structure of the network, as well as the method with which the probabilities are calculated using Bayesian probabilities. The implementation of this methodology is then demonstrated, with regards to data requirements and potential issues with the model, as well as the structure and calculation of the probability.

2. Literature Review

There exist many, varied methods of calculating the probabilities of complex systems such as the one seen in this project. For the purpose of this research, four commonly used tools for determining probability were considered and are discussed below. These methods were chosen to be examined based on literature detailing previous study done in similar fields. Two probabilistic methods, Bayesian networks and logistic regression, and two non-probabilistic methods, fuzzy logic and neural networks, are discussed and compared below to determine which gives the best representation of the system involved in this project, and thus the best outcome.

2.1 Bayesian Networks

Bayesian techniques are based on Baye's Law which relates the probability of event a occurring given event b also occurs to the probability that event b occurs given a has occurred, all in the context of background information, I. That is,

$$p(a|b,I) = \frac{p(b|a,I)p(a,I)}{p(b,I)}$$

Bayesian networks are a way of graphically representing conditional dependence between random variables, and are analysed using probabilistic methods; specifically, using Bayesian probabilities. The variables are represented as vertices in a directed acyclic graph (DAG), with the edges of the graph designating the dependence between the nodes. As Bayesian networks are a probabilistic technique of analysis, they will generally be a more accurate measure than possibilistic methods, such as fuzzy logic discussed below [5]. Bayesian inference easily accounts for subjectivity and conditionality [6], and it is possible to, without losing context, deconstruct the system in order to analyse certain sections [4]. However, like most probabilistic methods, Bayesian networks are computationally heavy, although they tend to be less so than other probabilistic methods [2] [5]. When used subjectively, they also require a large amount of cognitive work.

2.2 Logistic Regression

Logistic regression uses Frequentist probabilistic methods to predict the probability of a dependent variable taking a certain value based on a series of independent variables. Like Bayesian networks, it can be assumed that this method has greater accuracy than others discussed based on the fact it is probabilistic [5]. In addition, it is capable of easily incorporating interdependence between independent variables. However, the formulation of a logistic regression model requires a certain amount of holistic sample data, which is not necessarily readily available. Also as aforementioned, logistic regression is reasonably computationally heavy [2] [5]. This is a frequentist method, and as such uses the frequency interpretation of probability. Thus logistic regression is less facilitating when accounting for expert opinion and subjectivity than methods that incorporate conditional probability and Bayesian probabilities [6].

2.3 Fuzzy Logic

Fuzzy logic is a non-binary, many-valued logic where truth values can fall anywhere in the closed interval [0,1]. Its form of reasoning makes it simple to translate linguistic concepts into quantitative values, and it is a good method for sampling data. As a non-probabilistic method, fuzzy logic requires less computation than both Bayesian networks and logistic regression [5], and is also efficient in approximating functions [3]. However, it must be taken into account that as a possibilistic method, fuzzy logic is a much less accurate tool of measurement than probabilistic methods, such as the two discussed above [5].

2.4 Artificial Neural Networks

An artificial neural network is a method based on the neurological nervous system [7], involving a series of 'neurons', which represent variables, connected to each other. Neural networks differ to the other methods considered in that they easily incorporate adaptive learning, whereas this is not a major trait of the others, or necessarily possible [7]. Because of the way they are structured, they are able to derive complex relationships between variables [8]. Neural networks, however, use unpredictable procedures. This is in contrast to the other methods suggested, which are entirely predictable. It is not always possible to see how neural networks arrive at a conclusion. They also have a tendency to over fit the data, and are a computational burden [8].

2.5 Application

This particular project involves the prediction of success of a given complex mission. The data involved will not be holistic for the most part and potentially highly subjective in some areas. Because of these reasons, the method chosen to apply to this project is a Bayesian network. Bayesian inference accounts more easily for subjectivity than Frequentist methods such as logistic regression. In addition, the ability of a Bayesian network to break a system down and evaluate sections without losing context means it is appropriate for non-holistic data, such as what may be provided in this project. While the fuzzy logic method is computationally the most efficient, and also handles inconsistent data well, it tends to be less accurate than the probabilistic method. Bayesian networks have been known to be used in similar situations in the past, and are generally recommended as the appropriate statistical method for decision making [2]. The table below shows an overview of the abilities of each of the methods described in this section.

Table 1 Comparison of abilities of the four methods described

	Bayesian networks	Logistic regression	Fuzzy logic	Artificial neural networks
Probabilistic	x	x		
Accuracy	x	x		x
Ability to incorporate subjectivity	x		x	x
Ability to incorporate non-holistic data	x		x	x
Computationally inexpensive			x	
Predictable procedures	x	x	x	

3. Methodology

3.1 Development of the Bayesian network structure

The first stage in developing a Bayesian network is to consider the events relevant to the calculation of the probability, or probabilities, in question. By definition, a Bayesian network should be a complete representation of the system in which the events being measured probabilistically lie. The method for determining what is relevant depends on the situation, and in particular the data available. If a large amount of good data is available for a particular event, it may be possible to statistically assess the interdependency between events. Bayesian networks are able to account easily for any subjectivity, which is a useful trait when not enough data is available to assess quantitatively the likelihood of dependence.

The dependence links generated from the first step are represented in a directed acyclic graph (DAG), which forms the Bayesian network. Vertices in the said graph represent the events of the system, and are otherwise referred to as nodes. The directed edges connecting the nodes are dependent relationships between events, with edges running from cause to effect in most cases. Events which cause others are referred to as parents, while the events that they cause are known as children. The Bayesian network for this project, the development of which is discussed in Section 4, can be found in Appendix A.

3.2 Calculation of the probability

Not all of the events in this network will necessarily be variable. Some are fixed, and either determined by the person calculating the probability in question, in this case the probability of success, or by the environment in which they occur. Let $F = \{F_1, F_2, \dots, F_m\}$ be a set of fixed events, for m events. $F \subseteq U$ for the universal set U of all events in the system. It should be noted that on the diagram in Appendix A, parents of fixed nodes have not been shown. This is for the sole purpose of simplifying the diagram, although it should be noted that a complete Bayesian network would normally display these links as well. However, the removal of these links has no effect on the eventual probability calculated as no information can flow through fixed nodes from their parents. As these events are known to be fixed, their parental links are not shown in the diagram.

Alternatively, those events that are not fixed are designated variable events. Let V be the set of $n-1$ variable events, such that $V = F^C = \{V_1, V_2, \dots, V_{n-1}\}$. V includes events such as weather, and threats. Variable events are shown on the diagram in Appendix A as orange.

Let S be the event the probability of which is being calculated. In this case, S is the event that the mission succeeds.

As aforementioned, this project aims to calculate the probability of mission success, S, given certain fixed events, F_i. This is expressed as:

$$P(S|F) = P(S|F_1 \cap F_2 \cap \dots \cap F_m) \qquad \forall F_i \in F, i = 1, \dots, m.$$

The law of total probability states that:

$$P(S|F) = \sum_V P(S|V,F)P(V|F).$$

Let X_i be any variable event including S, for $X_i = 1, \dots, n$. It is now possible to expand the above equation in the following manner:

$$P(S|F) = \sum_V \prod_{i=1}^{n} P(X_i \,|X_1, \dots, X_{i-1}, X_{i+1}, \dots, X_n, F) \qquad (1)$$

By applying the chain rule for Bayesian networks to equation (1) can be simplified so it only depends on the parents of X_i [9].

$$P(S|F) = \sum_V \prod_{i=1}^{n} P(X_i \,|P_i) \qquad (2)$$

where $P_i \subseteq U$ is the set of all parents of X_i. Equation (2) is the general form of the equation used to calculate the probability of mission success in this project.

Bayesian networks require a fair amount of computation, as can be inferred from the general equation. To calculate the probability of n nodes, with each node i having a sample space of k_i, the number of terms t in the summation would equal:

$$t = \prod_{i=1}^{n} k_i$$

If every node could only take on two possible values, the number of iterations any program would have to do to calculate this summation would be 2^n, and it would be unrealistic to assume that each node will have a sample space of two. While Bayesian networks do reduce the amount of calculations necessary for a probabilistic method, there are still issues with the computation required for this particular method.

For each variable event there are a set of probability distributions, based on its parents. For example, take a node C that has two parents A and B as follows:

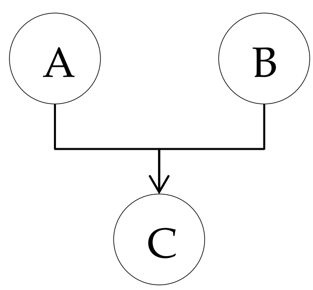

Figure 1 *Diagram of a simple Bayesian network with two parents*

Suppose A can take possible values {1,2}, B can take {1,2,3} and C {1,2}. The probability distributions for $P(C = c|A = a, B = b)$ for $a \in A, b \in B, c \in C$ are represented in tables a n example is shown in Table 2.

Table 2 Probability distributions of C given A and B

A=1	B=1	B=2	B=3
C=1	0.1	0.2	0.3
C=2	0.9	0.8	0.7

A=2	B=1	B=2	B=3
C=1	0.4	0.5	0.6
C=2	0.6	0.5	0.4

For example, from this it can be determined that $P(C = 1|A = 2, B = 1) = 0.4$. There must be a probability distribution for C given all combinations of a and b. Note that to be a valid probability distribution, the individual probabilities must satisfy:

$$\sum_{c \in C} P(C = c|A = a, B = b) = 1 \qquad \forall a \in A, b \in B$$

For a certain $a \in A, b \in B, c \in C$, an element in the matrix will represent $P(C = c|A = a, B = b)$.

However, one may want to calculate $P(C = c)$ regardless of the values of A and B. To do so, the probability distributions of A and B are required. For example:

Table 3 Probability distributions of A and B

A=1	0.1
A=2	0.9

B=1	0.6
B=2	0.3
B=3	0.1

From this we can calculate:

$$P(C = 1) = \sum_{a=1}^{2} \sum_{b=1}^{3} P(C = 1|A = a, B = b)P(A = a)P(B = b) = 0.42$$

This example demonstrates the basic method with which the more complex network for this project is constructed and analysed.

4. Application of method

4.1 Construction of the diagram

For the commencement of this project, the majority of dependence links were determined with expert opinion. The activities the weapon undergoes throughout the mission were derived from an existing model of the system. From that, possible influences on the various processes were considered. A chart was constructed with the activities that the weapon undergoes and their known influences listed. These activities and influences are represented in the nodes of the network. Each pairing of nodes was then considered in relation to each other, and dependence links were drawn from that process. Later on, new influences may be introduced and the probabilities required to populate the network will be generated more rigorously. However, in the first stage of developing the framework for mission success measurement it is not necessary to do so.

In this situation, the Bayesian network is structured as shown in Appendix A. This diagram was built in CORE®, a Model-Based Systems Engineering tool, and it displays two categories of events: activities and influences. The activities, represented by operational activities in CORE®, are the physical processes the weapon undertakes on the mission. The influences, represented by operational items in CORE®, are certain environmental effects and characteristics which are deemed to influence the mission. In general, influences will be the parents of activities, but as shown in the diagram some influences affect the probability of others.

4.2 Calculation of the probability

A Matlab® tool was developed to calculate the probability of mission success using a Bayesian network. The inputs of this function are the fixed events in F, and the output is the probability of mission success (MS) given these fixed events, or $P(MS|F)$. For this network, there are eight fixed events, which are to be decided before computing the probability. These include events such as target location and the time of launch. Further information on fixed and variable events can be found in Appendix A. By calculating the probability of success for various times, launch points and other controlled events, it is possible to optimise the result to find the values of the fixed events for which there is the greatest likelihood of success.

A separate function was created to demonstrate the calculation of the collateral damage estimate (CDE). This was done similarly to the method used to calculate mission success. However, the CDE demonstration gives a vector of the probability distribution, with each element corresponding to $P(TAI = x|F)$ for each $x \in TAI$, where TAI is the event that target area interaction is at a certain level x. The sum of these probabilities should equal one. As opposed to the calculation of probability of mission success, where collateral damage is assessed in a binary manner where the damage is either at an acceptable level or not. However, the function for the CDE has the potential to convey a more rigorous and

detailed assessment of the level of collateral damage expected, for example through detailed modelling and simulation.

4.3 Data requirements

What is necessary in terms of data for this project is a set of probability distributions for each variable event, given its parents. Whether that is found using quantitative methods, such as with experimental or simulation data, or whether it is determined via subjective means is beyond the scope of this part of the project. These probability distribution tables are input into Matlab® in the form of matrices of dimension $p + 1$, where p is the number of parents of a given event. Dummy probability distributions were constructed for the demonstration of this method.

In the demonstration provided, only simple, discrete variables were used for the example probability distributions. When the network is further developed, it is likely the system will incorporate continuous probability distributions. Discretisation of these variables is a method of working with the distributions on a practical level, but both fidelity and computational burden need to be considered when this process occurs.

4.4 Reduction of unnecessary computation

As mentioned earlier, a major practical issue with Bayesian networks is the computation required to generate probabilities, especially for complex networks. In order to minimise the computational time, the probability of all activities occurring before and including the initialisation of the weapon pre-separation was calculated separately. This has no effect on the overall probability of mission success, as there are no outside influences on this first stage of the weapon process and all events have binary outcomes of either success or failure. Similarly, mission success was calculated apart from the majority of the equation. This also has no effect on the outcome of the function. To reduce the number of iterations in the function, a method was devised to assess whether each product in the summation was equal to zero based on whether any of the individual probabilities calculated were equal to zero. These products are unnecessary for the summation, and thus were not computed.

4.5 AgenaRisk®

While it is possible to use Matlab® or other similar software in this situation, there exist several software packages specifically designed for the building and running of Bayesian networks. For this project, AgenaRisk® was used to reconstruct the network developed. This enabled relatively fast calculation of the probabilities for all nodes simultaneously. It also allowed for the calculation of probability propagation in the network. Running forward, the network provides a prediction of events of child nodes; while tracing from child nodes to parent nodes the network can be used for analysis of the influence of parent nodes. That is, if a probability is returned that is considered a poor result, there needs to be a method for analysing where in the system the problem, or problems, lie and thus if there is potential to improve the mission or model such that there is a higher likelihood of

success. Fixing the value of the event of mission success to an observed "false" in AgenaRisk® enables the user to observe the probability of all events occurring before that, given that the mission did not succeed. In doing so, it is possible to see the weak points in the system.

AgenaRisk® accommodates for continuous probability distributions by discretising the input values; the size of the intervals is determined by the user. Once again, the amount of computation required for any nodes with a large event space risks becoming an issue, but the ease with which the number of intervals can be changed ameliorates this problem somewhat. Figure 2 below shows a simple example of how nodes with normal distributions can be linked in AgenaRisk®.

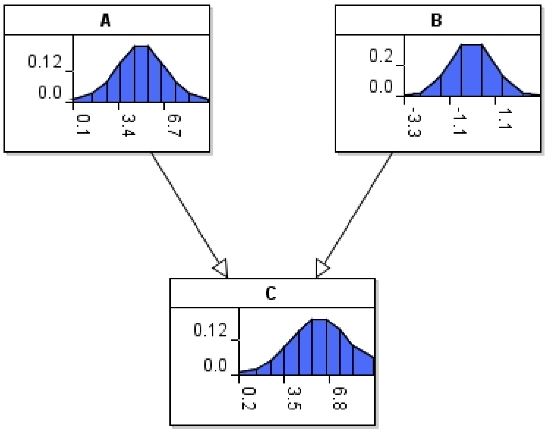

Figure 2 Diagram of a simple Bayesian network with continuous nodes

5. Conclusion

This report details the development of the framework for the statistical analysis of the probability of mission success. Firstly, based on the fact it is a probabilistic modelling tool easily able to account for subjectivity in data, a Bayesian network was chosen over other potentially valid but less well suited probabilistic and non-probabilistic methods. Given this, the structure of the Bayesian network in question was formulated using expert opinion to determine any dependent relationships and necessary influences for the system. A general formula for the calculation of the probability was derived and then implemented to demonstrate the method and the data necessary to generate the probability.

5.1 Future recommendations

The scope of this report covers only the beginning of the development of this statistical measurement tool. The next stage after this may be to start analysing in greater detail the data necessary to calculate mission success. In particular, the level of precision for each

distribution will need to be considered in regard to what lies in the event space for each node, or more specifically the size of the event space. One important factor to keep in consideration while doing this is the computational burden. On that topic, the further reduction of computation time is also something that should be considered later in this project. Doing so mathematically requires knowledge of the specific situation, and methods of reduction such as those discussed in the implementation will have to be determined on an individual basis. It would also be prudent to start collating the necessary data for measuring the likelihood of mission success. This data could be a combination of probability distributions calculated through Frequentist means, those calculated by more subjective methods, or both depending on the event in question.

To improve on the work currently completed, more appropriate software, in particular that aimed specifically at developing and calculating Bayesian networks such as AgenaRisk®, should be investigated. This may also help to solve the problem of computational burden as the software will generally incorporate algorithms to optimise the calculations, as well as creating a single location for all information regarding the network. Depending on the software, it may also assist in measuring posterior probabilities as mentioned previously. Software such as this may be used to validate tools developed separately for this project, or to validate relevant parts of those tools. Further exploration of the capabilities of AgenaRisk® is also recommended.

No method has been developed as of yet to measure the accuracy of the results obtained through this process. Without this, the tool that has been created will be less reliable. A method for calculating an estimate of accuracy would have to be implemented to be able to say with any statistical certainty how accurate the probability presented is, and this should be done in future to give the model a higher level of credibility.

While continuing this project, the framework of the Bayesian network should be put under revision at relevant stages. This includes ensuring all dependencies shown are relevant, potentially by more formally analysing the data using statistical methods, as mentioned in the report. It should also be remembered that not all the influences on the system have necessarily already been incorporated.

6. References

1. Broderick, N. et al. 2013, *Modelling requirements for mission success prediction*, MODSIM2013, Adelaide, Australia.

2. Das, B. 1999, *Representing uncertainties using Bayesian networks*, Technical report, DSTO, DSTO-TR-0918.

3. de Reus, N.M. 1994, *Assessment of benefits and drawbacks of using fuzzy logic, especially in fire control systems*, TNO Physics and Electronics Laboratory, Netherlands.

4. Falzon, L. 2005, *Using Bayesian network analysis to support centre of gravity analysis in military planning*, European Journal of Operational Research 170 (2), pp. 629-643.

5. Faragher, J. 2004, *Probabilistic methods for the quantification of uncertainty and error in computational fluid dynamics simulations*, Technical report, DSTO, DSTO-TR-1633.

6. Krieg, M.L. 2001, *A tutorial on Bayesian belief networks*, Technical note, DSTO, DSTO-TN-0403.

7. Stergiou, C. & Siganos, D. 1996, *Neural networks*, Imperial College London.

8. Tu, J.V. 1996, *Advantages and disadvantages of using artificial neural networks versus logistic regression for predicting medical outcomes*, Institute for Clinical Evaluative Sciences, Ontario.

9. Jensen, Finn V. *Bayesian Networks and Decision Graphs*, Springer-Verlag NY Inc, 2001.

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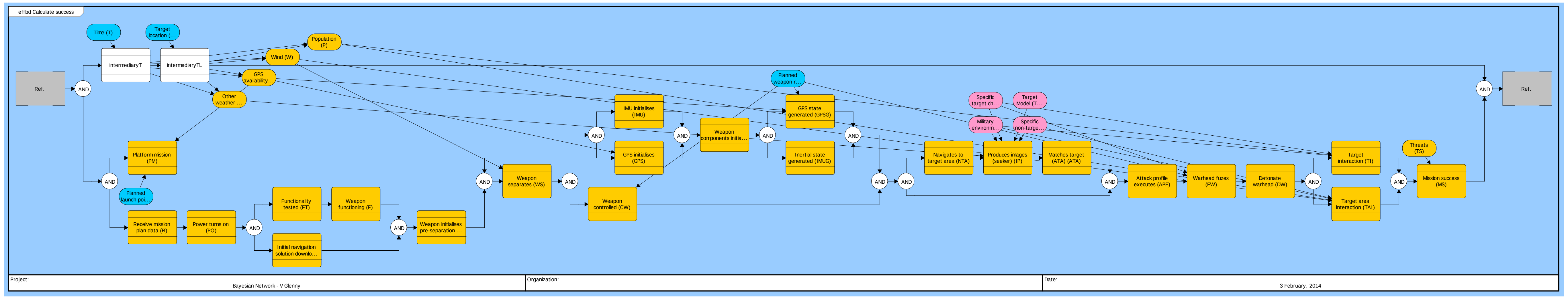
Appendix A: CORE® Diagram

This CORE® diagram represents the Bayesian network developed in this report. Each rectangular node is an activity that the weapon undergoes, and each oval node is an influence on the system.

The colours of the nodes represent the different data types, as follows:

- Orange: variable.
- Blue: fixed, and chosen when calculating the probability.
- Pink: fixed, but uncontrolled.



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